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The $\omega \rightarrow \rho\pi$ transition and $\omega \rightarrow 3\pi$ decay¹**J.L. Lucio M.⁽¹⁾, M. Napsuciale⁽¹⁾, M.D. Scadron⁽²⁾ and V.M. Villanueva⁽¹⁾.**

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Abstract.

We evaluate the $\omega \rightarrow \rho\pi$ transition and the $\omega \rightarrow 3\pi$ decay using a Quark Level Linear Sigma Model ($QL\sigma M$). We obtain $g_{\omega\rho\pi}^{QL\sigma M} = (10.33 - 14.75) \text{ GeV}^{-1}$ to be compared with other model dependent estimates averaging to $g_{\omega\rho\pi} = 16 \text{ GeV}^{-1}$. We show that in the $QL\sigma M$ a contact term is generated for the $\omega \rightarrow 3\pi$ decay. Although the contact contribution by itself is small, the interference effects turn out to be important.

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I. Introduction.

The problem of understanding low energy hadron dynamics is being supported by recent experimental data obtained at Novosibirsk-VEPP-2M detector and by the DAΦNE facility which will soon produce large amounts of experimental data in the energy region around 1 GeV. This region is particularly interesting since the nonperturbative QCD effects that govern hadron dynamics are far from being completely understood.

This paper is concerned with the $\omega \rightarrow \rho\pi$ vertex and $\omega \rightarrow \pi\pi\pi$ decay. Existing experimental data seems to confirm the Gell-Mann-Sharp-Wagner (GSW) suggestion [1] that the $\omega \rightarrow \pi\pi\pi$ decay is dominated by the $\omega\rho\pi$ transition followed by the $\rho \rightarrow \pi\pi$ decay, although a $\omega\pi\pi\pi$ contact contribution can not be excluded. The theoretical description of these problems have been considered by a number of authors using as different techniques as approximate SU(3) symmetry[2], Vector Meson Dominance (VMD)[3], QCD sum rules [4,5,6] and effective chiral lagrangians [7].

It is our purpose in this paper to consider the Quark Level Linear Sigma Model (QL σ M) predictions for the $\omega \rightarrow \rho\pi$ transition and the $\omega \rightarrow \pi\pi\pi$ decay. The model describes the U(2) \times U(2) chiral invariant interactions of effective quarks with pseudoscalar and scalar mesons. Vector mesons are incorporated in the model as gauge bosons, even though in our work they only appear as external particles. In this model, besides the GSW mechanism, the $\omega \rightarrow \pi\pi\pi$ decay proceeds through a quark box diagrams with the ω and three pions in the box vertices, which can be interpreted as a contact term. By itself the contact term is not important: it leads to a $\Gamma(\omega \rightarrow \pi\pi\pi) \cong 0.1$ MeV. However its interference with the amplitude arising from the GSW mechanism leads to a sizable 25% effect in the decay rate.

The paper is organized as follows: In section II we introduce the model and discuss the determination of the coupling constants entering in the Lagrangian. Then in section III we compute -working in the soft momentum limit for what follows- the QL σ M quark loop for $g_{\omega\rho\pi}$. In section IV we present calculations for $\omega \rightarrow \pi\pi\pi$ within QL σ M. This include the GSW mechanism and quark box contributions. Comparison with the observed $\omega \rightarrow \pi\pi\pi$ decay rate is then made. In section V we draw our conclusions.

II. The Model.

The quark level $L\sigma M$ describes the $U(2) \times U(2)$ chiral invariant interaction of mesons and effective quarks. We have chosen to work with a pseudoscalar rather than a derivative coupling which has the advantage that no anomalous interactions are required to describe one pion processes. Vector mesons are incorporated in the model as gauge bosons, even though in our work they only appear as external particles. The $QL\sigma M$ Lagrangian is:

$$\mathcal{L}_{int} = \bar{\psi}[i \not{D} - M + g(S + i\gamma_5 P)]\psi + (D_\mu B D^\mu B^\dagger)/2 + \dots \quad (1)$$

where $B \equiv S + iP$ with S, P scalar and pseudoscalar fields respectively ($P = \frac{1}{\sqrt{2}}(\eta_0 + \vec{\tau} \cdot \vec{\pi})$, scalar fields being defined in a similar way) and ψ denotes the quark isospinor. Vector fields are introduced as gauge fields through the covariant derivative:

$$\begin{aligned} D_\mu \psi &\equiv (\partial_\mu + ig_V V_\mu)\psi, \\ D_\mu B &\equiv \partial_\mu B + ig_V [V_\mu, B]. \end{aligned} \quad (2)$$

The quark mass matrix M in Eq.(1) is generated by spontaneous breaking of chiral symmetry, quark masses being related to the πqq coupling g through the Goldberger-Trieman relation (GTR) $m_q = gf_\pi$. The ellipsis in Eq.(1) refers to vector meson kinetic and mass terms and to scalar-pseudoscalar Yukawa interactions which are not relevant for the purposes of this work.

The interaction terms involved in the calculations presented in this paper are:

$$\mathcal{L}_{int} = g_V(\vec{\rho} \times \vec{\pi}) \cdot \partial \vec{\pi} - \frac{g_V}{2} \bar{\psi} \gamma^\mu \vec{\tau} \psi \cdot \vec{\rho}_\mu - \frac{g_V}{2} \bar{\psi} \gamma^\mu \psi \omega_\mu. \quad (3)$$

Thus, the model relates the ρqq and ωqq couplings ($\frac{g_V}{2}$) to the $\rho\pi\pi$ coupling (g_V). This constant can be estimated from experimental data on the $\rho \rightarrow \pi\pi$ decay. Another determination comes from Vector Meson Dominance as applied to the constituent quark level. The conventional VMD procedure leads to the following relations between the $\rho - \gamma$ ($f_{\rho\gamma}$), $\omega - \gamma$ ($f_{\omega\gamma}$), and the Vqq couplings:

$$g_{\rho qq} = \frac{em_\rho^2}{f_{\rho\gamma}}, \quad g_{\omega qq} = \frac{em_\omega^2}{3f_{\omega\gamma}}. \quad (4)$$

The $\frac{1}{3}$ factor in the last expression arises due to the fact that the isoscalar contribution to the quark electric charge is proportional to the quark baryonic number. Using the data [8] for the leptonic decays $\rho \rightarrow e^+e^-$, $\omega \rightarrow e^+e^-$ we obtain $g_V^{\rho ll} \equiv g_{\rho qq} = 5.03$ and $g_V^{\omega ll} \equiv g_{\omega qq} = 5.68$, whereas the $\rho \rightarrow \pi\pi$ decay leads to $g_V^{\rho\pi\pi} = 6.01$ (superindices in g_V indicates the process from which it is extracted).

Summarizing, for g_V in Eq.(2,3) we can use either $g_V^{\rho\pi\pi} = 6.01$, $g_V^{\rho ll} \equiv g_{\rho qq} = 5.03$ or $g_V^{\omega ll} \equiv g_{\omega qq} = 5.68$. In the rest of the paper we report numerical results for these three different values of g_V , even though we argue below that the more accurate determination comes from $g_V^{\omega ll}$ (besides being close to the average value of $g_V^{\rho\pi\pi}$ and $g_V^{\rho ll}$).

III. QL σ M and the $\omega \rightarrow \rho\pi$ transition.

It is conventional to define the $g_{\omega\rho\pi}$ coupling in terms of the amplitude:

$$M_{\omega\rho\pi} = g_{\omega\rho\pi} \epsilon_{\mu\nu\alpha\beta} P^k P'^\nu \epsilon^\alpha(\omega) \eta^\beta(\rho), \quad (5)$$

where $P(P')$ denote the $\omega(\rho)$ momentum and $\epsilon(\eta)$ are the respective polarization vectors. Although there is no phase space from which to measure an $\omega \rightarrow \rho\pi$ transition, this vertex can be extracted from many theoretical models. Very approximate SU(3) symmetry of the 1970's suggest [2] $g_{\omega\rho\pi} \approx 16 \text{ GeV}^{-1}$, while QCD sum rules obtain [4] $g_{\omega\rho\pi} \approx (15 \rightarrow 17) \text{ GeV}^{-1}$, and the analog light cone sum rules method extracts [6] $g_{\omega\rho\pi} = 15 \text{ GeV}^{-1}$. Recently QCD sum rules for the polarization operator in an external field concludes [5] $g_{\omega\rho\pi} \approx 16 \text{ GeV}^{-1}$. It is interesting to mention that this kind of coupling affects strongly the calculations for some other processes [9].

In the QL σ M the $\omega\rho\pi$ vertex is described in terms of the quark loops of Fig (1). A straightforward calculations yields:

$$\mathcal{M} [\omega(Q, \eta) \rightarrow \rho(k, \varepsilon) + \pi(r)] = g_{\omega\rho\pi} \epsilon(k, r, \eta, \varepsilon),$$

where

$$g_{\omega\rho\pi} = -2iN_c m_q g_V^2 g (I^a + I^b), \quad (6)$$

with

$$I^a \equiv \int \frac{d^4\ell}{(2\pi)^4} \cdot \frac{1}{\nabla(\ell)\nabla(\ell-k)\nabla(\ell-k-r)}, \quad (7)$$

$$I^b = I^a(r \leftrightarrow k)$$

where $\nabla(p) \equiv p^2 - m_q^2$. Since we are interested in the $\omega\rho\pi$ vertex involved in $\omega \rightarrow \pi\pi\pi$ decay, we calculate the integrals in Eq. (7) in the soft pion momentum limit, $k, r \rightarrow 0$. This amounts to take the leading term in an expansion of the amplitude in the external momenta k and r . In this approximation $I^a = I^b$ and we get for the $\omega\rho\pi$ coupling:

$$g_{\omega\rho\pi}^{QL\sigma M} = \frac{g_V^2 N_c}{8\pi^2 f_\pi}. \quad (8)$$

Using the values of the coupling constants as previously estimated we conclude:

$$g_{\omega\rho\pi}^{QL\sigma M} = (10.33, 13.19, 14.75) \text{ GeV}^{-1}. \quad (9)$$

These values are obtained using in Eq.(8) $g_V = (g_V^{\rho ll}, g_V^{\omega ll}, g_V^{\rho\pi\pi})$ respectively. Thus, $QL\sigma M$ predictions for $g_{\omega\rho\pi}$ are slightly smaller than other determinations [2-7].

As a byproduct of our analysis we report the predictions of the $QL\sigma M$ for $\omega \rightarrow \pi^0\gamma$ and $\rho^0 \rightarrow \pi^0\gamma$ which are very similar to the ones presented so far, the only difference being the appearance of the γqq coupling instead of the Vqq coupling. We obtain:

$$\mathcal{M}(V(Q, \eta) \rightarrow \pi(r)\gamma(k, \varepsilon)) = g_{V\pi\gamma}^{QL\sigma M} \epsilon(k, r, \eta, \varepsilon), \quad (10)$$

where

$$\begin{aligned} g_{\rho\pi\gamma}^{QL\sigma M} &= -i \frac{g_V e}{8\pi^2} \frac{g}{m_q} N_c (e_u + e_d), \\ g_{\omega\pi\gamma}^{QL\sigma M} &= -i \frac{g_V e}{8\pi^2} \frac{g}{m_q} N_c (e_u - e_d). \end{aligned} \quad (11)$$

A worth noticing point is that both $g_{\omega\rho\pi}$ and $g_{V\pi\gamma}$ as calculated in this work, agree with those derived from a chiral lagrangian with vector mesons in the hidden scheme [10].

Using the GTR and $g_V = g_V^{\rho ll}, g_V^{\omega ll}, g_V^{\rho\pi\pi}$ we obtain respectively:

$$\begin{aligned}
|g_{\omega\pi\gamma}^{QL\sigma M}| &= (0.622, 0.703, 0.743)GeV^{-1}, \\
|g_{\rho\pi\gamma}^{QL\sigma M}| &= (0.207, 0.234, 0.247)GeV^{-1}.
\end{aligned} \tag{12}$$

These numbers are to be compared with the experimental results $|g_{\omega\pi\gamma}^{exp}| = 0.703 \pm 0.020 \text{ GeV}^{-1}$, $|g_{\rho\pi\gamma}^{exp}| = 0.29 \pm 0.037 \text{ GeV}^{-1}$.

Notice that the $QL\sigma M$ predictions for the $\omega \rightarrow \pi\gamma$ transition are in good agreement with the experimental data when $g_V = g_V^{\omega ll}$ - as extracted from the ω leptonic width - is used. On the other hand predictions for $\rho \rightarrow \pi\gamma$ agree with experimental results within two standard deviations. These results and the fact that the measurements for the $\omega \rightarrow \pi\gamma$ decay rate are more accurate than for $\rho \rightarrow \pi\gamma$ lead us to consider $g_V = 5.68$ as the most confident value for g_V .

IV. $QL\sigma M$ AND THE $\omega \rightarrow \pi\pi\pi$ DECAY.

In this section we work out the $QL\sigma M$ predictions for the $\omega \rightarrow \pi\pi\pi$ decay. Two mechanisms contribute to this process within the model. The first one is through an intermediate ρ in the s, t, u channels as depicted in Fig(2), which involve the previously calculated $g_{\omega\rho\pi}$ and $g_{\rho\pi\pi}$. The second mechanism involves quark boxes, as shown in Fig (3).

The ρ -mediated contribution leads to a $\omega \rightarrow \pi\pi\pi$ amplitude:

$$\mathcal{M}^{GSW} [\omega(Q, \eta) \rightarrow \pi^+(q)\pi^-(p)\pi^0(r)] = A^{GSW} \varepsilon(\eta, p, q, r), \tag{13}$$

where:

$$A^{GSW} = 2g_{\omega\rho\pi} g_V \left(\frac{1}{s - m_\rho^2} + \frac{1}{t - m_\rho^2} + \frac{1}{u - m_\rho^2} \right). \tag{14}$$

The amplitude corresponding to the quark boxes contribution is:

$$A^{box} = -4g_V g^3 m_q I \varepsilon(\eta, p, q, r) \tag{15}$$

where

$$I = \sum_{i=1}^6 I^i,$$

and

$$I^1 = \int \frac{d^4\ell}{(2\pi)^4} \cdot [\nabla(\ell)\nabla(\ell-r)\nabla(\ell-r-p)\nabla(\ell-r-p-q)]^{-1}. \quad (16)$$

The 5 remaining integrals are obtained from I^1 by permutations in Eq. (16) of the pion momenta (p, q, r) . We again consider the leading term of an expansion of this integral in the pions momenta, since higher order terms are expected to be suppressed by powers of $\left(\frac{m_\pi}{m_q}\right)^2$. In this approximation we obtain:

$$A^{box} = -\frac{g_V}{4\pi^2 f_\pi^3}. \quad (17)$$

The decay rate $\Gamma(\omega \rightarrow \pi\pi\pi)$ is given by:

$$\Gamma(\omega \rightarrow 3\pi) = \frac{g_V^2 g_{\omega\rho\pi}^2 m_\omega^3}{768\pi^3} J, \quad (18)$$

where J stands for the phase space integral

$$J = \int_{4\beta}^{(1-\beta)^2} dx \int_{y_+}^{y_-} dy |f(x, y)|^2 \Delta(x, y), \quad (19)$$

with

$$y_{\pm} = \frac{1}{2}(1 + 3\beta - x \mp \sqrt{(1 - \frac{4\beta}{x})(x - (1 + \sqrt{\beta})^2)(x - (1 - \sqrt{\beta})^2)}), \quad (20)$$

and

$$f(x, y) = f^{GSW}(x, y) + f^{box}(x, y)$$

,
where

$$\begin{aligned} f^{GSW}(x, y) &= \frac{1}{x - \alpha} + \frac{1}{y - \alpha} + \frac{1}{1 + 3\beta - x - y - \alpha}, \\ f^{box}(x, y) &= -\frac{m_\omega^2}{24\pi^2 f_\pi^3 g_{\omega\rho\pi}}. \end{aligned}$$

The Kibble determinant Δ in Eq.(19) is given by:

$$\Delta(x, y) = xy(1 + 3\beta) - x^2y - xy^2 - \beta(1 - \beta)^2, \quad (21)$$

where

$$x = \frac{s}{m_\omega^2}, \quad y = \frac{t}{m_\omega^2}, \quad \beta = \frac{m_\pi^2}{m_\omega^2}, \quad \alpha = \frac{m_\rho^2}{m_\omega^2}.$$

The phase space integral has been worked out by Thews [3] in the case of a constant matrix element. Since the numerical evaluation of the integral is straightforward, below we report the results according to the exact numerical evaluation, which of course reproduce the results of Ref. [3] in the case of a constant matrix element.

The quark box contribution to the $\omega \rightarrow \pi\pi\pi$ decay rate is small:

$$\Gamma_{box}(\omega \rightarrow 3\pi) \simeq 0.11 \text{ MeV}. \quad (22)$$

In table 3 we summarize the numerical results for the $\omega \rightarrow 3\pi$ decay width for the three possible values of g_V .

Table I

g_ρ	$g_{\omega\rho\pi}(GeV^{-1})$	$\Gamma^{GSW}(\omega \rightarrow 3\pi)(MeV)$	$\Gamma^{TOT}(\omega \rightarrow 3\pi)(MeV)$
5.03	10.33	2.72	3.79
5.68	13.19	5.66	7.39
6.01	14.75	7.92	10.06

These results have to be compared with the experimental data:

$$\Gamma_{exp}(\omega \rightarrow 3\pi) = 7.5 \pm 0.1 MeV \quad (23)$$

Notice that while the quark box contribution alone is negligible, the interference with the ρ mediated amplitude is important.

In the $QL\sigma M$ the non-resonant contribution arising from quark box diagrams (Fig.3) necessarily exist. From the numerical results reported in Table 3 we conclude that $QL\sigma M$ predictions are in agreement with experimental

data within the 25% uncertainty associated with the determination of g_V . It is worth remarking that the most clean extraction of this coupling (from ω leptonic width) is close to the experimental result.

V. SUMMARY.

We worked out the quark level Linear Sigma Model predictions for the $\omega\rho\pi$ coupling constant. We obtain $g_{\omega\rho\pi}^{QL\sigma M} = (10.33, 13.19, 14.75)GeV^{-1}$. These values correspond to g_V as extracted from ρ leptonic width, ω leptonic width and $\rho \rightarrow \pi\pi$ respectively. The $QL\sigma M$ predictions for $g_{\omega\rho\pi}$ are smaller than other determinations [2-7].

The non-resonant contribution to the $\omega \rightarrow 3\pi$ decay naturally arise within the $QL\sigma M$. We evaluated the boxes contributions (Fig. (3)) to the $\omega \rightarrow 3\pi$ decay. We find an amplitude which leads to a small decay rate by itself. However, the interference of the boxes contribution with the ρ mediated amplitude lead to a sizable (25 – 30%) effect in the decay rate.

The amplitudes for the $\omega \rightarrow \pi\gamma$ ($g_{\omega\pi\gamma}$) and $\rho \rightarrow \pi\gamma$ ($g_{\rho\pi\gamma}$) can be obtained as a byproduct of the $g_{\omega\rho\pi}$ calculation. The agreement with experimental data is good in the case of the $\omega \rightarrow \pi\gamma$ decay. In the case of the $\rho \rightarrow \pi\gamma$ decay; QL σ M predictions agree with experimental data within two standard deviations.

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Figure captions:

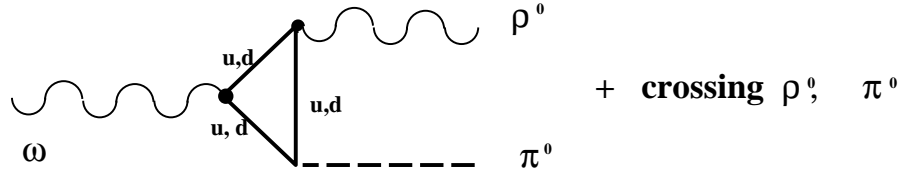


Fig. 1
Quark loop triangles contribution for $\omega \rightarrow \rho^0 \pi^0$ transition.

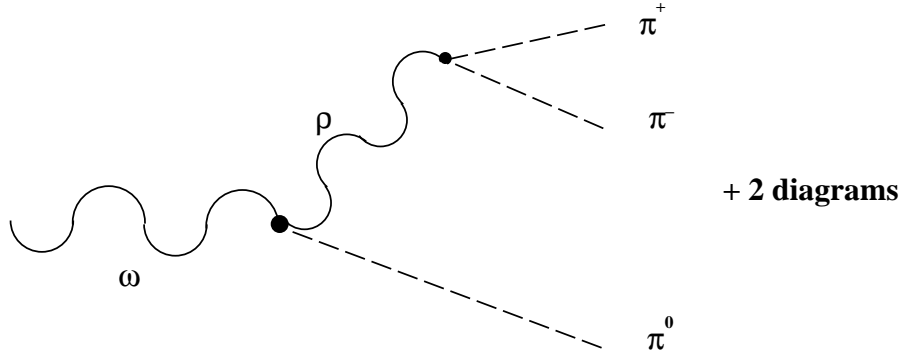


Fig. 2
Intermediate ρ contributions for the $\omega \rightarrow \pi^0 \pi^+ \pi^-$ decay.

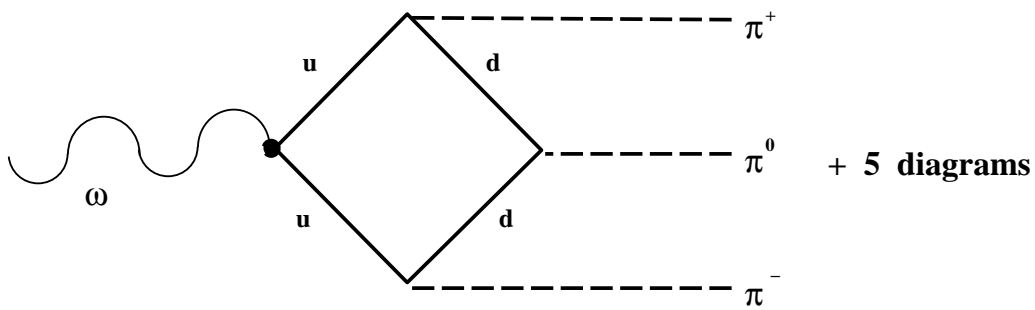


Fig. 3
 Quark boxes for $\omega \rightarrow \pi^+ \pi^0 \pi^-$ decay.